1. *Engineering Structures and Materials*

1.1 Introduction

Mechanics of materials is a branch of applied mechanics that deals with the behavior of solid bodies subjected to various types of loading. This field of study is known by several names, including "Strength of materials" and "mechanics of deformable bodies." The solid bodies considered in this book include axially loaded members, shafts in torsion, thin shells, beams, and columns, as well as structures that are assemblies of these components. Usually the objectives of our analysis will be the determination of the stresses, strains, and deflections produced by the loads. If these quantities can be found for all values of load up to the failure load, then we will have a complete picture of the mechanical behavior of the body.

A thorough understanding of mechanical behavior is essential for the safe design of all structures, whether buildings and bridges, machines and motors, submarines and ships, or airplanes and antennas. Hence, mechanics of materials is a basic subject in many engineering fields. Of course, statics and dynamics are also essential, but they deal primarily with the forces and motions associated with particles and rigid bodies. In mechanics of materials, we go one step further by examining the stresses and strains that occur inside real bodies that deform under loads. We use the physical properties of the materials (obtained from experiments) as well as numerous theoretical laws and concepts, which are explained in succeeding sections of this book.

Theoretical analyses and experimental results have equally important roles in the study of mechanics of materials. On many occasions, we will make logical derivations to obtain formulas and equations for predicting mechanical behavior, but we must recognize that these formulas cannot be used in a realistic way unless certain properties of the materials are known. These properties are available to us only after suitable experiments have been carried out in the laboratory. Also, because many practical problems of great importance in engineering cannot be handled efficiently by theoretical means, experimental measurements become a necessity.

The historical development of mechanics of materials is a fascinating blend of both theory and experiment; experiments have pointed the way to useful results in some instances, and theory has done so in others. Such famous men as Leonardo da Vinci (1452-1519) and Galileo Galilei (1564-1642) performed experiments to determine the strength of wires, bars, and beams, although they did not develop any adequate theories (by today's standards) to explain their test results. Such theories came much later. By contrast, the famous mathematician Leonhard Euler (1707-1783) developed the mathematical theory of columns and calculated the theoretical critical load of a column in 1744, long before any experimental evidence existed to show the significance of his results. Thus, for want of appropriate tests, Euler's results remained unused for many years, although today they form the basis of column theory.

When studying mechanics of materials from this book, you will find that your efforts are divided naturally into two parts: first, understanding the logical development of the concepts, and second, applying those concepts to practical situations. The former is accomplished by studying the derivations, discussions, and examples, and the latter by solving problems. Some of the examples and problems are numerical in character, and others are algebraic tor symbolic). An advantage of numerical problems is that the magnitudes of all quantities are evident at every stage of the calculations. Sometimes these values are needed to ensure that practical limits (such as allowable stresses) are not exceeded. Algebraic solutions have certain advantages, too. Because they lead to formulas, algebraic solutions make clear the variables that affect the final result. For instance, a certain quantity may actually cancel out of the solution, a fact that would not be evident from a numerical problem. Also apparent in algebraic solutions is the manner in which variables affect the results, such as the appearance of one variable in the numerator and another in the denominator. Furthermore, a symbolic solution provides the opportunity to check the dimensions at any stage of the work. Finally, the most important reason for obtaining an algebraic solution is to obtain a general formula that can be programmed on a computer and used for many different problems. In contrast, a numerical solution applies to only one set of circumstances. Of course, you must be adept at both kinds of solutions, hence you will find a mixture of numerical and algebraic problems throughout the book.

Numerical problems require that you work with specific units of measurements. The two accepted standards of measurement are the. International System of Units (SI) and the U.S. Customary System (USCS). As you know significant digits are very important in engineering. In our work in this section, *three significant digits* provides enough accuracy.

1.2 Normal Stress and Strain

The fundamental concepts of stress and strain can be illustrated by considering a prismatic bar that is loaded by axial forces P at the ends, as shown in Figure 1. A ***prismatic bar*** is a straight structural member having constant cross section throughout its length. In this illustration, the axial forces produce a uniform stretching of the bar; hence, the bar is said to be in ***tension.***

To investigate the internal stresses produced in the bar by the axial forces, we make an imaginary cut at section *aa* (Figure 1). This section is taken perpendicular to the longitudinal axis of the bar; hence, it is known as a ***cross section***. We now isolate the part of the bar to the right of the cut as a free body. The tensile load P acts at the righthand end of the free body; at the other end are forces representing the action of the removed part of the bar upon the part that remains. These forces are continuously distributed over the cross section, analogous to the continuous distribution of hydrostatic pressure over a submerged horizontal surface. The intensity of force (that is, the force per unit area) is called the ***stress*** and is commonly denoted by the Greek letter σ (sigma). Assuming that the stress has a uniform distribution over the cross section (see Figure 1), we can readily see that its resultant is equal to the intensity σ times the

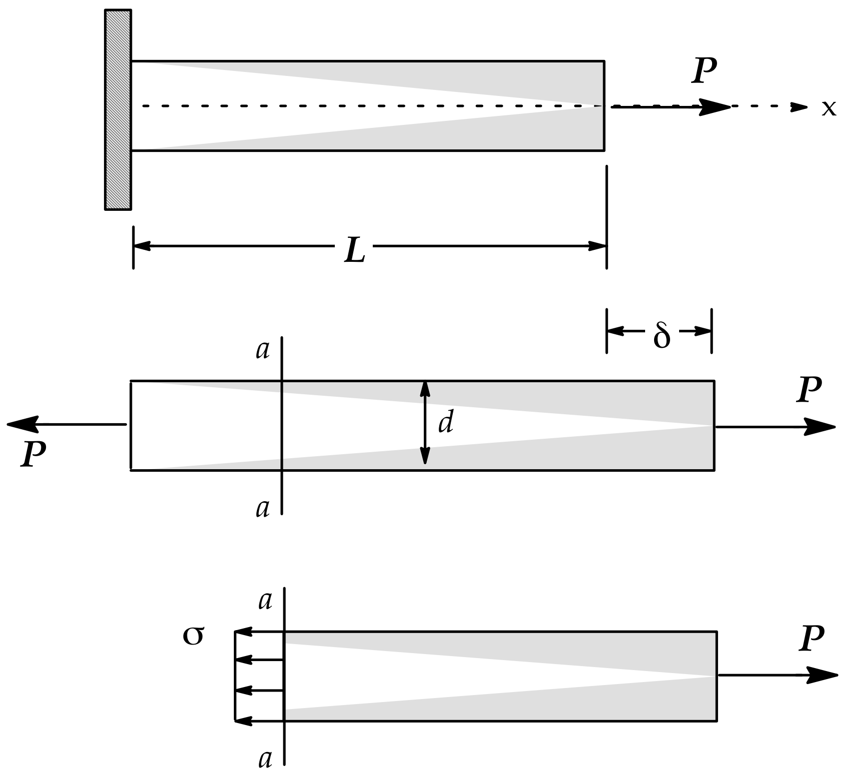


Figure 1. Bar in tension.

cross-sectional area A of the bar. Furthermore, from the equilibrium (balancing of forces) of the body shown in Figure 1, it is also evident that this resultant must be equal in magnitude and opposite in direction to the applied load P. Hence, we obtain

$$\boxed{\sigma = \frac{P}{A}} \tag{1–1}$$

as the equation for the uniform stress in an axially loaded, prismatic bar of arbitrary cross-sectional shape. When the bar is stretched by the forces P, as shown in the figure, the resulting stresses are ***tensile stresses***; if the forces are reversed in direction, causing the bar to be compressed, we obtain ***compressive stresses***. Inasmuch as the stress a acts in a direction perpendicular to the cut surface, it is referred to as a ***normal stress***. Thus, normal stresses may be either tensile or compressive stresses. Later, we will encounter another type of stress, called a ***shear stress***, that acts parallel to the surface.

When a ***sign convention*** for normal stresses is required, it is customary to define tensile stresses as positive (+) and compressive stresses as negative (–).

Because the normal stress σ is obtained by dividing the axial force by the cross-sectional area, it has ***units*** of force per unit of area. When SI units are used, force is expressed in newtons (N) and area in square meters (m^2). Hence, stress has units of newtons per square meter

(N/m^2), or pascals (Pa). However, the pascal is such a small unit of stress that it is necessary to work with large multiples. To illustrate this point, we have only to note that it takes almost 7000 pascals to make 1 psi.

As an example, a typical tensile stress in a steel bar might have a magnitude of 140 megapascals (140 MPa), which is 140×10^6 pascals. Other units that may be convenient to use are the kilopascal (kPa) and gigapascal (GPa); the former equals 10^3 pascals and the latter equals 10^9 pascals. Although it is not recommended in SI, you will sometimes find stress given in newtons per square millimeter (N/mm^2), which is a unit identical to the megapascal (MPa).

When using USCS units, stress is customarily expressed in pounds per square inch (psi) or kips per square inch (ksi). One kip, or kilopound, equals 1000 pounds. For instance, a typical stress in a steel bar might be 20,000 psi or 20 ksi.

In order for the equation $\sigma = P/A$ to be valid, the stress σ must be uniformly distributed over the cross section of the bar. This condition is realized if the axial force P acts through the centroid of the crosssectional area. When the load P does not act at the centroid, bending of the bar will result, and a more complicated analysis is necessary (you will learn more in CIVL 3322 Strength of Materials). However, we will assume throughout our discussions that all axial forces are applied at the centroid of the cross section unless specifically stated otherwise.

The uniform stress condition pictured in Figure 1 exists throughout the length of the member except near the ends. The stress distribution at the ends of the bar depends upon the details of how the axial load P is actually applied. If the load itself is distributed uniformly over the end, then the stress pattern at the end will be the same as elsewhere. However, the load is usually concentrated over a small area, resulting in high localized stresses and nonuniform stress distributions over cross sections in the vicinity of the load. As we move away from the ends, the stress distribution gradually approaches the uniform distribution.

It is usually safe to assume that the formula $\sigma = P/A$ may be used with good accuracy at any point within the bar that is at least a distance d away from the ends, where d is the largest transverse dimension of the bar (see Figure 1). Of course, even when the stress is not uniform, the equation $\sigma = P/A$ will give the ***average normal stress***.

An axially loaded bar undergoes a change in length, becoming longer when in tension and shorter when in compression. The total change in length is denoted by the Greek letter δ (delta) and is pictured in Figure 1 for a bar in tension. This elongation is the cumulative result of the stretching of the material throughout the length L of the bar. Let us now assume that the material is the same everywhere in the bar. Then, if we consider half of the bar, it will have an elongation equal to $\delta/2$; similarly, if we consider a unit length of the bar, elongation equal to $1/L$ times the total elongation δ. In this manner, we arrive at the concept of elongation per unit length, or ***strain***, denoted by the Greek letter ε (epsilon) and given by the equation

$$\boxed{\epsilon = \frac{\delta}{L}} \qquad (1\text{–}2)$$

If the bar is in tension, the strain is called a ***tensile strain***, representing an elongation or stretching of the material. If the bar is in compression, the strain is a ***compressive strain*** and the bar shortens. Tensile strain is taken as positive (+), and compressive strain as negative (–). The strain ε is called a ***normal strain*** because it is associated with normal stresses.

Because normal strain ε is the ratio of two lengths, it is a ***dimensionless quantity***; that is, it has no units. Thus, strain is expressed as a pure number, independent of any system of units. Numerical values of strain are usually very small, especially for structural materials, which ordinarily undergo only small changes in dimensions.

As an example, consider a steel bar having length L of 2.0 m. When loaded in tension, the bar might elongate by an amount δ equal to 1.4 mm. The corresponding strain is

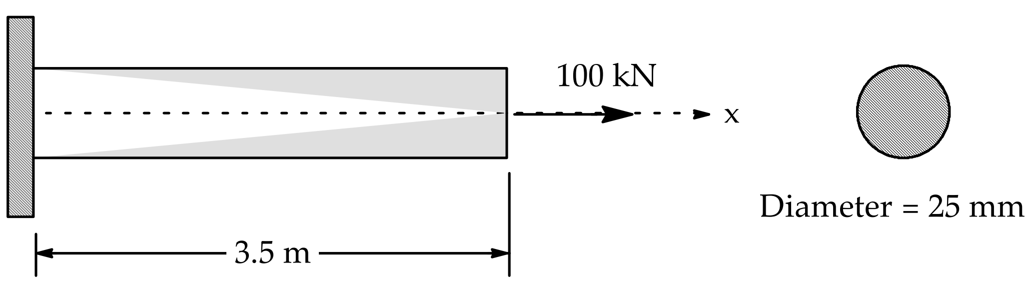
$$\epsilon = \frac{\delta}{L} = \frac{1.4 \times 10^{-3} m}{2.0 m} = 0.0007 = 700 \times 10^{-6}$$

In practice, the original units of δ and L are sometimes attached to the strain itself, and then the strain is recorded in forms such as mm/m, μm/m, and in./in. For instance, the strain ε in the preceding illustration could be given as 700 μm/m or 700 x 10^{-6} in,/in.

The definitions of normal stress and strain are based upon purely statical and geometrical considerations, hence Equations (1–1) and (1–2) can be used for loads of any magnitude and for any material. The principal requirement is that the deformation of the bar be uniform, which in turn requires that the bar be prismatic, the loads act through the centroids of the cross sections, and the material be ***homogeneous*** (that is, the same throughout all parts of the bar).

The resulting state of stress and strain is called ***uniaxial stress and strain***. Further discussions of uniaxial stress, including stresses and strains in other than the longitudinal direction of the bar, are given in later sections. We will also encounter more complicated stress states, such as biaxial stress and plane stress, in later chapters.

Example – A prismatic bar with a circular cross section is subjected to an axial tensile force. The measured elongation is δ = 1.5 mm. Calculate the tensile stress and strain in the bar.



Assuming the axial force act at the centroid of the end cross section, then the stress is

$$\sigma = \frac{P}{A} = \frac{100kN}{\frac{\pi(25mm)^2}{4}} = 203.718327\ N/mm^2 = 204\ MPa$$

The strain is

$$\epsilon = \frac{\delta}{L} = \frac{1.5mm}{3.5m} = 429 \times 10^{-6}$$

1.3 Stress-Strain Diagrams

The mechanical properties of materials used in engineering are determined by tests performed on small specimens of the material. The tests are conducted in materials-testing laboratories equipped with testing machines capable of loading the specimens in a variety of ways, including static and dynamic loading in tension and compression.

In order that test results may be compared easily, the dimensions of test specimens and the methods of applying loads have been standardized. One of the major standards organizations is the American Society for Testing and Materials (ASTM), a national technical society that publishes specifications and standards for materials and testing. Other standardizing organizations are the American Standards Association (ASA) and the National Bureau of Standards (NBS).

The most common materials test is the ***tension test***, in which tensile loads are applied to a cylindrical specimen. The ends of the specimen are enlarged where they fit in the grips so that failure will occur in the central uniform region, where the stress is easy to calculate, rather than near the ends, where the stress distribution is complicated. An ***extensometer*** is used to measures the elongation during loading.

The ASTM standard tension specimen has a diameter of 0.5 in. and a ***gage length*** of 2.0 in. between the gage marks, which are the points where the extensometer arms are attached to the specimen. As the specimen is pulled, the load P is measured and recorded, either automatically or by reading from a dial. The elongation over the gage length is measured simultaneously with the load, usually by mechanical gages, although electric-resistance strain gages are also used. In a ***static test***, the load is applied very slowly; however, in a ***dynamic test***, the rate of loading may be very high and also must be measured because it affects the properties of the materials.

The axial stress σ in the test specimen is calculated by dividing the load P by the cross-sectional area A (see Equation (1–1)). When the initial area of the bar is used in this calculation, the resulting stress is called the ***nominal stress*** (other names are conventional stress and engineering stress). A more exact value of the axial stress, known as the ***true stress***, can be calculated by using the actual area of the bar, which can become significantly less than the initial area for some materials.

The average axial strain in the bar is found from the measured elongation δ between the gage marks by dividing δ by the gage length L, Equation (1–2). If the initial gage length is used (for instance, 2.0 in.), then the ***nominal strain*** is obtained. Of course, the distance between the gage marks increases as the tensile load is applied. If the actual distance is used in calculating the strain, we obtain the ***true strain***, or ***natural strain***.

Compression tests of metals are customarily made on small specimens in the shape of cubes or circular cylinders. Cubes are often 2.0 in. on a side, and cylinders usually have diameters of about 1 in. with lengths of 1 to 12 in. Both the load applied by the machine and the shortening of the specimen may be measured. The shortening should be measured over a gage length that is less than the total length of the specimen in order to eliminate end effects.

Concrete is tested in compression on every important construction project to ensure that the required strengths have been obtained. The standard ASTM concrete test specimen is 6 in. in diameter, 12 in. long, and 28 days old (the age of concrete is important because concrete gains strength as it cures).

After performing a tension or compression test and determining the stress and strain at various magnitudes of the load, we can plot a diagram of stress versus strain. Such a ***stress-strain diagram*** is characteristic of the material and conveys important information about the mechanical properties and type of behavior. Stress-strain diagrams were originated by Jacob Bernoulli (1654-1705) and J. V. Poncelet (1788-1867).

The first material we will discuss is ***structural steel***, also known as mild steel or low-carbon steel. Structural steel is one of the most widely used metals, being the principal steel used in buildings, bridges, towers, and many other types of construction. A stress-strain diagram for a typical structural steel in tension is shown in Figure 2 (not to scale).

Strains are plotted on the horizontal axis and stresses on the vertical axis. The diagram begins with a straight line from O to A. In this region, the stress and strain are directly proportional, and the behavior of the material is said to be ***linear elastic***. Beyond point A, the linear relationship between stress and strain no longer exists; hence, the stress at A is called the ***proportional limit***. For low-carbon steels, this limit is in the range 30 to 40 ksi, but high-strength steels (with higher carbon content plus other alloys) can have proportional limits of 80 ksi and more.

With an increase in the load beyond the proportional limit, the strain begins to increase more rapidly for each increment in stress. The stress-strain curve then has a smaller and smaller slope, until, at point B, the curve becomes horizontal. Beginning at this point, considerable elongation occurs, with no noticeable increase in the tensile force (from B to C on the diagram). This phenomenon is known as ***yielding*** of the material, and the stress at point B is called the ***yield stress***, or ***yield point***. In the region from B to C, the material becomes ***perfectly plastic***, which means that it can deform without an increase in the applied load. The elongation of a mild-steel specimen in the perfectly plastic region is typically 10 to 15 times the elongation that occurs between the onset of loading and the proportional limit.

After undergoing the large strains that occur during yielding in the region BC, the steel begins to ***strain harden***. During strain hardening, the material undergoes changes in its atom-

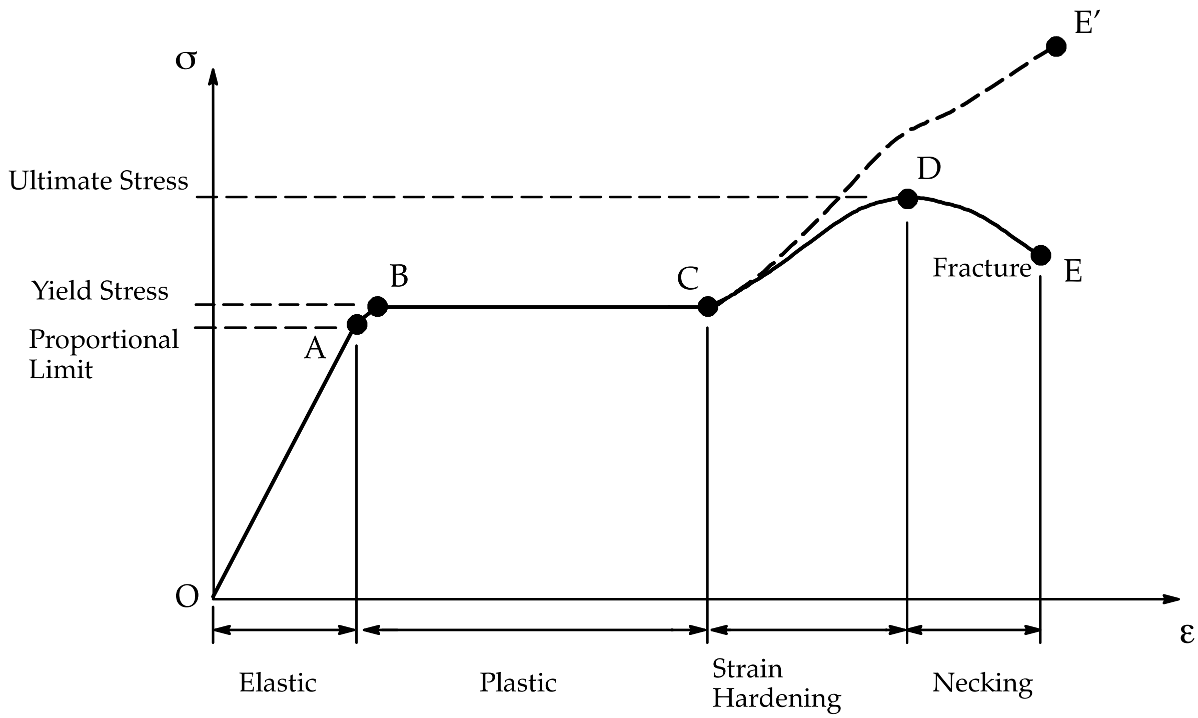


Figure 2. Stress–strain diagram for structural steel.

ic and crystalline structure, resulting in increased resistance of the material to further deformation. Thus, additional elongation requires an increase in the tensile load, and the stress-strain diagram has a positive slope from C to D The load eventually reaches its maximum value, and the corresponding stress tat point D) is called the ***ultimate stress***. Further stretching of the bar is actually accompanied by a reduction in the load, and ***fracture*** finally occurs at a point such as E on the diagram.

Lateral contraction of the specimen occurs when it is stretched, resulting in a decrease in the cross-sectional area, as previously mentioned. The reduction in area is too small to have a noticeable effect on the calculated value of stress up to about point C, but beyond that point the reduction begins to alter the shape of the diagram. Of course, the true stress is larger than the nominal stress because it is calculated with a smaller area.

In the vicinity of the ultimate stress, the reduction in area of the bar becomes clearly visible and a pronounced ***necking*** of the bar occurs. If the actual cross-sectional area at the narrow part of the neck is used to calculate the stress, the ***true stress-strain curve*** will follow the dashed line CE′ in Figure 2. The total load the bar can carry does indeed diminish after the ultimate stress is reached (curve DE), but this reduction is due to the decrease in area of the bar and not to a loss in strength of the material itself.

In reality, the material withstands an increase in stress up to failure (point E′). For most practical purposes, however, the conventional stress-strain curve OABCDE, which is based upon the original cross-sectional area of the specimen and hence is easy to calculate, provides satisfactory information for use in design.

The diagram in Figure 2 shows the general characteristics of the stress-strain curve for mild steel, but its proportions are not realistic because, as already mentioned, the strain that occurs from B to C may be 15 times the strain occurring from O to A. Furthermore, the strains from C to E are many times greater than those from B to C. Figure shows a stress-strain diagram for mild steel drawn approximately to scale. In this figure, the strains from O to A are so small in comparison to the strains from A to E that they cannot be seen, and the linear part of the diagram appears to be a vertical line.

The presence of a pronounced yield point followed by large plastic strains is an important characteristic of mild steel that is sometimes utilized in practical design. Materials that undergo large strains before failure are classified as ***ductile***. An advantage of ductility is that visible distortions may occur if the loads become too large, thus providing an opportunity to take remedial action before an actual fracture occurs. Also, ductile materials are capable of absorbing large amounts of energy prior to fracture. Ductile materials include mild steel, aluminum and some of its alloys, copper, magnesium, lead, molybdenum, nickel, brass, bronze, monel metal, nylon, teflon, and many others.

Structural steel contains about 0.2% carbon as an alloy and is classified as a low-carbon steel. With increasing carbon content, steel becomes less ductile but has a higher yield stress

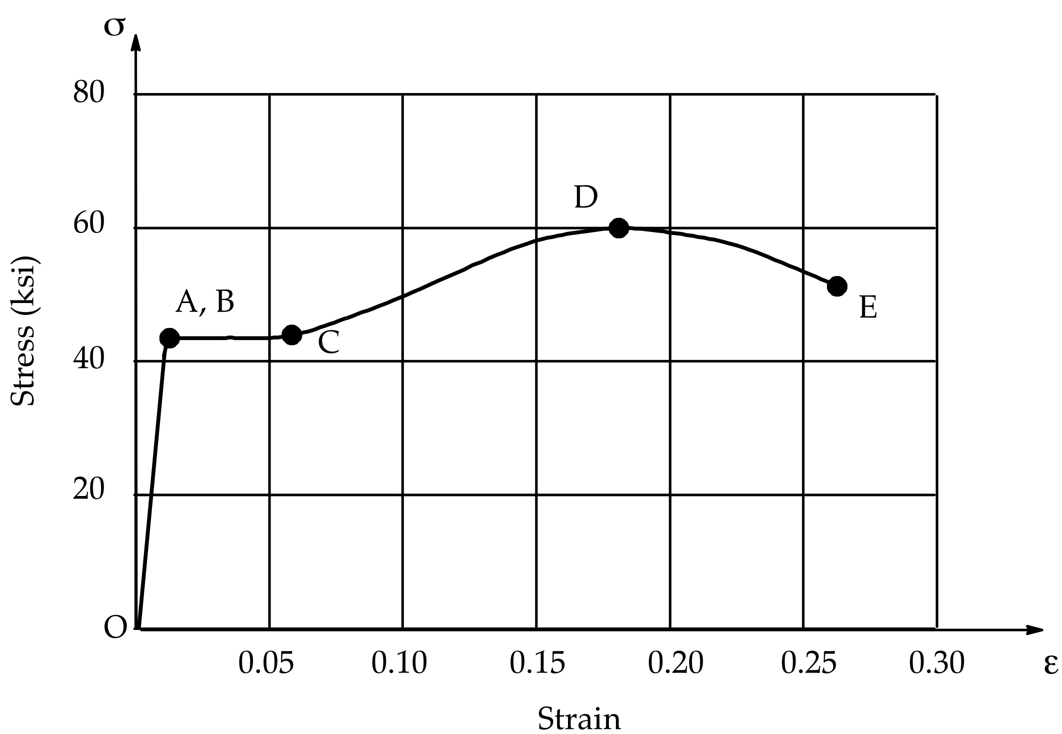


Figure 3. Stress–strain diagram for structural steel in tension.

and higher ultimate stress. The physical properties of steel are also affected by heat treating, the presence of other alloys, and manufacturing processes such as rolling.

Many ***aluminum alloys*** possess considerable ductility, although they do not have a clearly definable yield point. Instead, they exhibit a gradual transition from the linear to the nonlinear region, as shown by the stress-strain diagram in Figure 4. Aluminum alloys suitable for structural purposes are available with proportional limits in the range 10 to 60 ksi and ultimate stresses in the range 20 to 80 ksi.

When a material such as aluminum does not have an obvious yield point and yet undergoes large strains after the proportional limit is exceeded, an arbitrary yield stress may be determined by the ***offset method***. A line is drawn on the stress-strain diagram parallel to the initial linear part of the curve but is offset by some standard amount of strain, such as 0.002 tor 0.2%). The intersection of the offset line and the stress-strain curve (point A in the figure) defines the yield stress.

Since this stress is determined by an arbitrary rule and is not an inherent physical property of the material, it should be referred to as the ***offset yield stress***. For a material such as aluminum, the offset yield stress is slightly above the proportional limit. In the case of structural steel, with its abrupt transition from the linear region to the region of plastic stretching, the offset stress is essentially the same as both the yield stress and the proportional limit.

Rubber maintains a linear relationship between stress and strain up to very large strains in the vicinity of 0.1 or 0.2. The behavior after the proportional limit is exceeded depends upon the type of rubber (see Figure 5). Some kinds of soft rubber continue to stretch enormously without failure. The material eventually offers increasing resistance to the load, and the stress-strain curve turns markedly upward prior to failure. You can easily sense this characteristic behavior by stretching a rubber band.

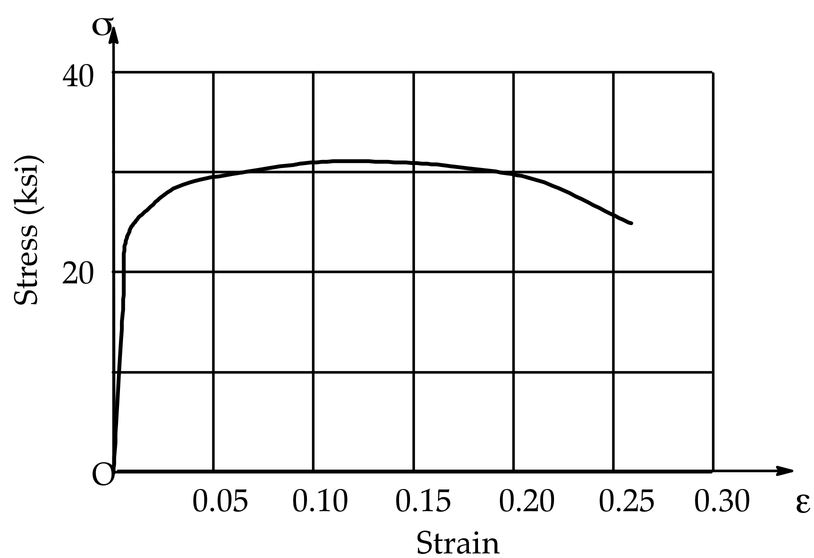


Figure 4. Stress–strain diagram for aluminum in tension.

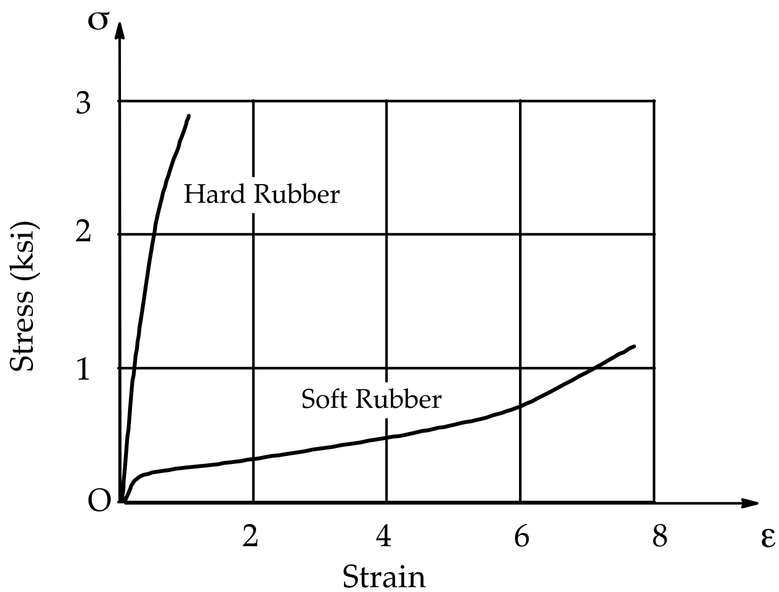


Figure 5. Stress–strain diagram for aluminum in tension.

The ductility of a material in tension can be characterized by its elongation and by the reduction in area at the cross section where fracture occurs. The ***percent elongation*** is defined as follows:

$$\textit{Percent elongation} = \frac{L_f - L_o}{L_o}(100\%) \tag{1–3}$$

in which L_0 is the original gage length and L_f is the distance between the gage marks at fracture. Because the elongation is not uniform over the length of the specimen but is concentrated in the region of necking, the percent elongation depends upon the gage length. Therefore, when stating the percent elongation, the gage length should also be given. For a 2 in. gage length, steel may have an elongation in the range of 10% to 40%, depending upon composition; for structural steel, values of 25% or 30% are common. In the case of aluminum alloys, the elongation varies from 1% to 45%, depending upon composition and treatment.

The ***percent reduction in area*** measures the amount of necking that occurs and is defined as follows:

$$\textit{Percent reduction in area} = \frac{A_o - A_f}{A_o}(100\%) \tag{1–4}$$

in which A_o, is the original cross-sectional area and A_f, is the final area at the fracture section. For ductile steels, the reduction is about 50%.

Materials that fail in tension at relatively low values of strain are classified as ***brittle*** materials. Examples are *concrete*, stone, cast iron, glass, ceramic materials, and many common metallic alloys. These materials fail with only little elongation after the proportional limit (point A in Figure 6) is exceeded, and the fracture stress (point B) is the same as the ultimate stress.

High-carbon steels behave in a brittle manner; they may have a very high yield stress (over 100 ksi in some cases), but fracture occurs at an elongation of only a few percent.

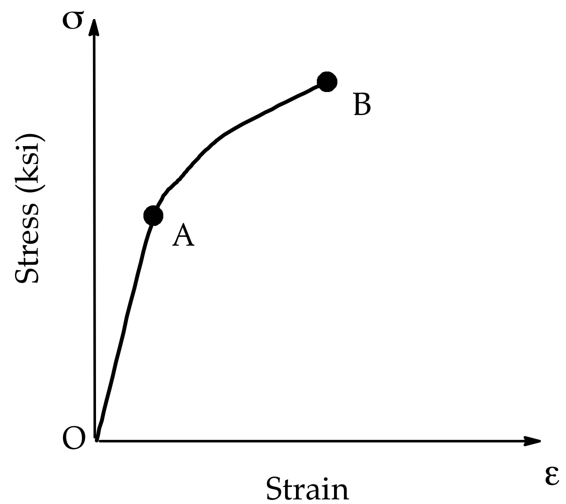


Figure 6. Stress–strain diagram for a brittle material.

Ordinary ***glass*** is a nearly ideal brittle material, because it exhibits almost no ductility whatsoever. The stress-strain curve for glass in tension is essentially a straight line, with failure occurring before any yielding takes place. The ultimate stress is about 10,000 psi for certain kinds of plate glass, but great variation exists, depending upon the type of glass, size of specimen, and the presence of microscopic defects. Glass fibers can develop enormous strengths, and ultimate stresses over 1,000 ksi have been attained.

Stress-strain diagrams for ***compression*** have different shapes from those for tension. Ductile metals such as steel, aluminum, and copper have proportional limits in compression very close to those in tension, hence the initial regions of their compression stress-strain diagrams are very similar to the tension diagrams. However, when yielding begins, the behavior is quite different. In a tension test, the specimen is being stretched, necking may occur, and ultimately fracture takes place. When a small specimen of ductile material is compressed, it begins to bulge outward on the sides and become barrel shaped. With increasing load, the specimen is flattened out, thus offering increased resistance to further shortening (which means the stress-strain curve goes upward). These characteristics are illustrated in Figure 7, which shows a compression stress-strain diagram for copper.

Brittle materials in compression typically have an initial linear region followed by a region in which the shortening increases at a higher rate than does the load. Thus, the compression stress-strain diagram has a shape that is similar to the shape of the tensile diagram. However, brittle materials usually reach much higher ultimate stresses in compression than in tension. Also, unlike ductile materials in compression (see Figure 6), brittle materials actually fracture or break at the maximum load. The tension and compression stress-strain diagrams for a par-

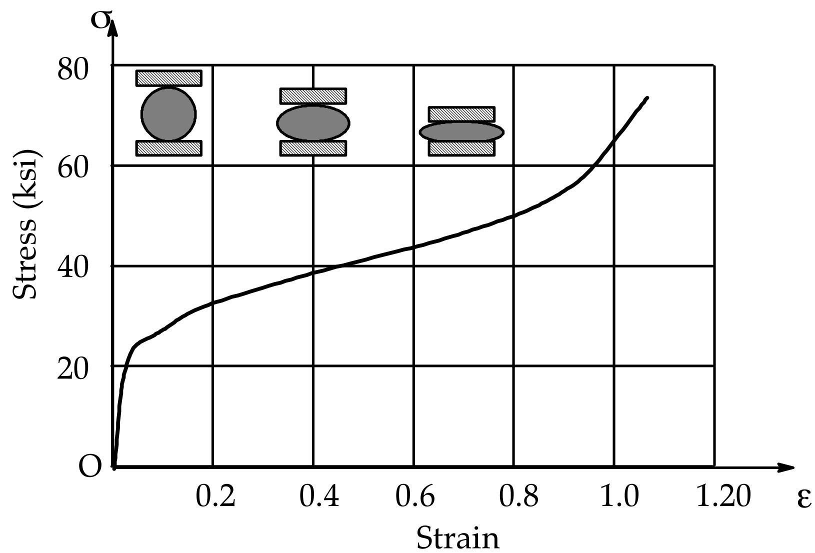


Figure 7. Compression stress-strain diagram for copper.

ticular type of cast iron are given in Figure 8. Curves for other brittle materials, such as concrete and stone, have similar shapes but quite different numerical values.

A table of important ***mechanical properties*** for various materials can be found in most Strength of Materials textbooks. However, properties and stress-strain curves vary greatly, even for the same material, because of different manufacturing processes, chemical composition, internal defects, temperature, and many other factors. Hence, any data obtained from general tables should be considered as typical, but not necessarily suitable for a specific application.

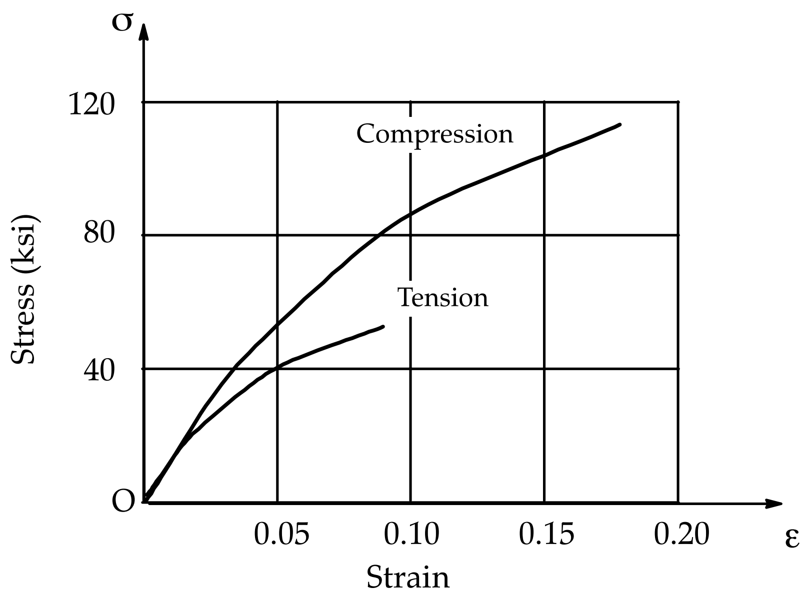


Figure 8. Stress–strain diagram for cast iron.

1.4 Elasticity and Plasticity

The stress-strain diagrams described in the preceding section illustrate the behavior of various materials as they are ***loaded*** statically in tension or compression. Now let us consider what happens when the load is slowly removed, and the material is ***unloaded***. Assume, for instance, that we apply a load to a tensile specimen so that the stress and strain go from O to A on the stress-strain curve in Figure 9. Suppose further that, when the load is removed, the material follows exactly the same curve back to the origin O. This property of a material, by which it returns to its original dimensions during unloading, is called ***elasticity***, and the material itself is said to be ***elastic***. Note that the stress-strain curve from O to A need not be linear in order for the material to be elastic

Now let us suppose that we load this same material to a much higher level, so that point B is reached on the stress-strain diagram, see Figure 9. In this case, when unloading occurs, the material follows line BC on the diagram. This unloading line typically is parallel to the initial portion of the loading curve; that is line BC is parallel to a tangent to the stress-strain curve at O. When point C is reached, the load has been entirely removed, but a ***residual strain***, or ***permanent strain***, OC remains in the material. The corresponding residual elongation of the bar is called the ***permanent set***. Of the total strain OD developed during loading from O to B, the strain CD has been recovered elastically and the strain OC remains as a permanent strain. Thus, during unloading the bar returns partially to its original shape; hence, the material is said to be ***partially elastic***.

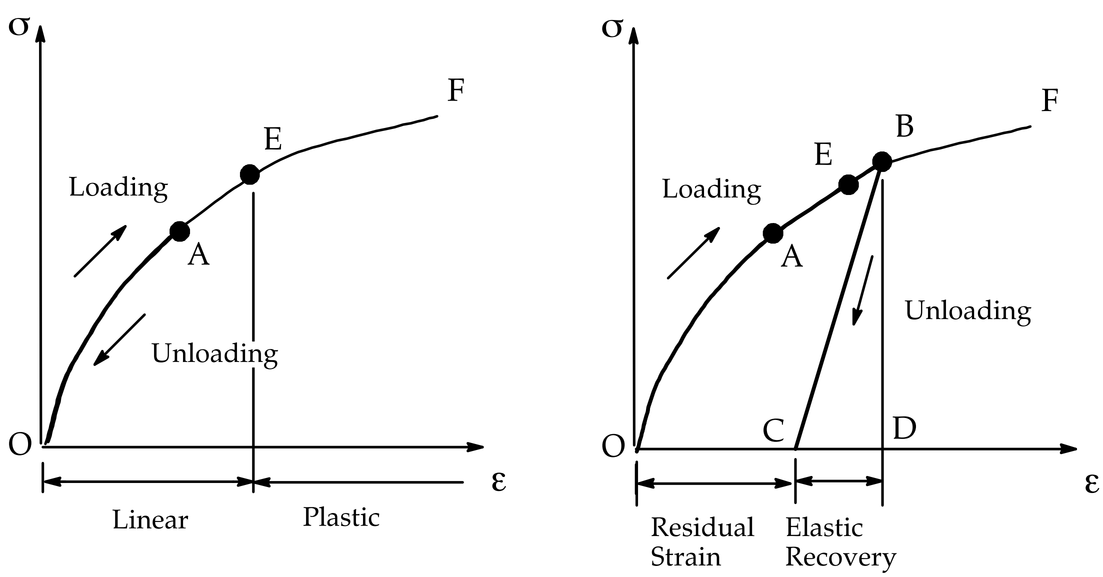


Figure 9. Elastic behavior; partially elastic behavior.

When a bar is being tested, the load can be increased from zero to some small selected value and then removed. If there is no permanent set (that is, if the elongation of the bar returns to zero) then the material is elastic up to the stress represented by the selected value of the load. This process of loading and unloading can be repeated for successively higher values of load.

Eventually, a stress will be reached such that not all the strain is recovered during unloading. By this procedure, it is possible to determine the stress at the upper limit of the elastic region; for instance, it could be the stress at point E in Figure 9. This stress is known as the ***elastic limit*** of the material.

Many materials, including most metals, have linear regions at the beginning of their stress-strain curves (see Figures 2 and 4). As explained in a previous section, the upper limit of this linear region is defined by the proportional limit. Usually the elastic limit is slightly above, or nearly the same as, the proportional limit. Hence, for many materials the two limits are assigned the same numerical value. In the case of mild steel, the yield stress is also very close to the proportional limit, so that for practical purposes the yield stress, the elastic limit, and the proportional limit are assumed to be equal. Of course, this situation does not hold for all materials. Rubber provides the outstanding example of a material that is elastic far beyond the proportional limit.

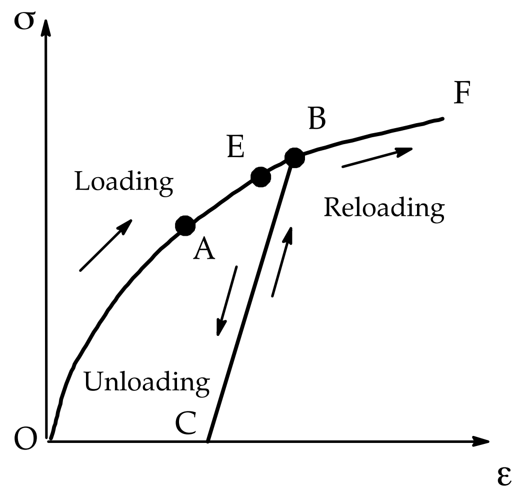


Figure 10. Reloading of a material and raising the yield stress.

The characteristic of a material by which it undergoes inelastic strains beyond those at the elastic limit is known as ***plasticity***. Thus, on the stress-strain curve in Figure 9, we have an elastic region followed by a plastic region. When large deformations occur in a ductile material loaded into the plastic region, the material is said to undergo ***plastic flow***.

If the material remains within the elastic range, it can be loaded, unloaded, and loaded again without significantly changing the behavior. However, when loaded into the plastic range, the internal structure of the material is altered and its properties change. For instance, we have already observed that a permanent strain exists in the specimen after unloading from the plastic region (Figure 9).

Now suppose that the material is reloaded after such an unloading (Figure 10). The new loading begins at point C on the diagram and continues upward to B, the point at which unloading began during the first loading cycle. The material then follows the original stress-strain diagram toward point F. During the second loading, the material behaves in a linear manner from C to B, hence the material has a higher proportional limit and a higher yield stress than before. Thus, by stretching a material, it is possible to raise the yield point, although the ductility is reduced because the amount of yielding from B to F is less than from E to F.

The stress-strain diagrams previously described are obtained from tension tests involving only static loading of the specimens; hence, the passage of time did not enter into our discussions. However, some materials develop additional strains over long periods of time and are said to ***creep***. This phenomenon can manifest itself in a variety of ways. For instance, let us suppose that a vertical bar (Figure 11) is loaded by a constant force *P*. When the load is applied initially, the bar elongates by an amount δ_0. Let us assume that this loading and the corresponding elongation take place during a time interval of duration t_0. Subsequent to time t_0, the load remains constant. However, due to creep, the bar may gradually lengthen, as shown in Figure 11, even though the load does not change. This behavior occurs with many materials, although sometimes the change is too small to be of concern.

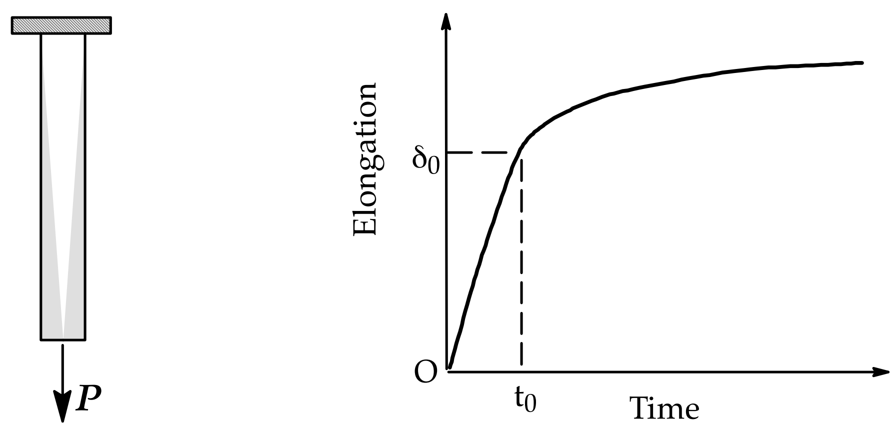


Figure 11. Creep in a bar under constant load.

As a second example of creep, consider a wire that is stretched between two immovable supports so that it has an initial tension stress σ_0, (Figure). Again, we will denote the time during which the wire is loaded initially as t_0 (Figure 12). With the elapse of time, the stress in the wire gradually diminishes, eventually reaching a constant value, even though the supports at the ends of the wire do not move. This process, which is a manifestation of creep, is called ***relaxation*** of the material.

Creep is usually more important at high temperatures than at ordinary temperatures; hence, it must be considered in the design of engines, furnaces, and other structures that operate at elevated temperatures for long periods of time. However, materials such as steel, concrete, and wood creep slightly even at atmospheric temperatures. Therefore, it is sometimes necessary to compensate for creep effects in ordinary structures.

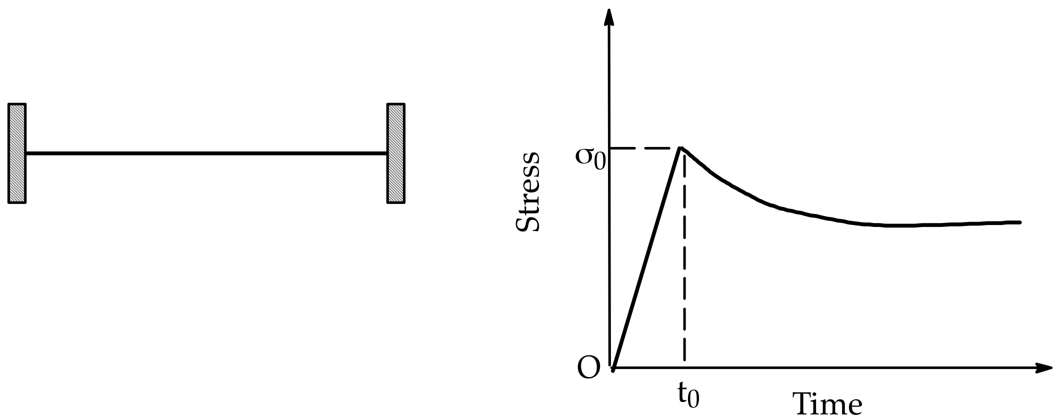


Figure 12. Relaxation of stress in a wire under constant strain.

For example, creep of concrete can create “waves” in bridge decks because of sagging between the supports. One remedy is to construct the deck with an upward ***camber***, which is an initial deflection above the horizontal, so that, when creep occurs, the spans lower to the level position.

1.5 Linear Elasticity and Hooke’s Law

Most structural materials have an initial region on the stress-strain diagram in which the material behaves both elastically and linearly. An example is the region from the origin O up to the proportional limit at point A on the stress-strain curve for structural steel (see Figure 2). Other examples are the regions below both the proportional limits and the elastic limits on the diagrams of Figures 4 through 8. When a material behaves elastically and also exhibits a linear relationship between stress and strain, it is said to be ***linearly elastic***. This type of behavior is extremely important in engineering because many structures and machines are designed to function at low levels of stress in order to avoid permanent deformations from yielding or plastic flow. Linear elasticity is a property of many solid materials, including metals, wood, concrete, plastics, and ceramics.

The linear relationship between stress and strain for a bar in simple tension or compression can be expressed by the equation

$$\boxed{\sigma = E\epsilon} \tag{1–5}$$

in which E is a constant of proportionality known as the ***modulus of elasticity*** for the material. The modulus of elasticity is the slope of the stress-strain diagram in the linearly elastic region, and its value depends upon the particular material being used. The units of E are the same as the units of stress, inasmuch as strain is dimensionless. Hence, the units of E are psi or ksi in USCS units and pascals in SI units.

The equation $\sigma = E\varepsilon$ commonly known as ***Hooke's law***, for the famous English scientist Robert Hooke (1635-1703). Hooke was the first person to investigate the elastic properties of materials, and he tested such diverse materials as metal, wood, stone, bones, and sinews. He measured the stretching of long wires supporting weights and observed that the elongations "always bear the same proportions one to the other that the weights do that make them." Thus, Hooke established the linear relationship between the applied load and the resulting elongation.

Equation (1-5) applies only to ordinary tension and compression; for more complicated states of stress, a generalized Hooke's law is required. In calculations, tensile stress and strain are usually considered as positive, and compressive stress and strain as negative.

The modulus of elasticity E has relatively large values for materials that are very stiff, such as structural metals. Steel has a modulus of approximately 30,000 ksi or 200 GPa; for aluminum, E equals approximately 10,600ksi or 70 GPa. More flexible materials have a lower modulus; a typical value for wood is 1,600 ksi or 11 GPa. Representative values of E are typically listed in most Strength of Materials textbooks. For most materials, the value of E in compression is the same as in tension.

The modulus of elasticity is often called ***Youug's modulus***, after another English scientist, Thomas Young (1773-1829). In connection with an investigation of tension and compression of prismatic bars, Young introduced the idea of a "modulus of the elasticity." However, his modulus was not the same as the one in use today, because it involved properties of the bar as well as of the material.